# Logic and Cognitive Science

Igor Sedlár[1] and Ján Šefránek[2]

**Abstract.** Our aim is to show that the logical point of view and methods of logic are indispensable for the understanding of human cognition. However, the results of some well known psychological experiments may be seen as denying the relevance of logic in studying human reasoning. Wrong design decisions and interpretations of these experiments are analyzed in this chapter. Arguments supporting an externalist position for a level of descriptions of cognition are presented. Finally, relations of logic and actual human reasoning are analyzed and illustrated on some examples.

## 1 Introduction

Our goal is to argue that logic is relevant for cognitive science and that the contribution of logic to the understanding of human cognition is fundamental.

Knowledge and reasoning are essential capabilities and results of human cognition. Our approach is based on an externalist viewpoint. According to this viewpoint, knowledge and reasoning can be studied as objective phenomena, independent on neural and mental processes.

We will analyze two psychological experiments that can be seen as implying that logic is not relevant for understanding of human reasoning, viz. the selection task and the suppression task. We shall argue that these experiments are based on misleading design decisions and the interpretation of their results aimed against the relevance of logic is not justified.

Subsequently, the 'logical point of view' is presented. We specify the relevant types of problems and the logical method for studying knowledge and reasoning.

After that, a variety of particular logical systems and ways of doing logic is described. An attempt to characterize cognitive tasks corresponding to different logical systems is presented.

## 2 Cognition and Truth

The goal of this section is to argue that important features of cognition and cognitive abilities are connected to the external environment. Most importantly, contents of sound cognitions are crucially dependent on the state of the external world. Consequently, knowledge and reasoning can (and should) be studied as objective phenomena, independent on neural and mental processes.

Some features of cognition are recognizable even on low biological levels. Cognitive biology considers the ability of living agents to distinguish on the molecular level, cell level and the level of simple organisms as an exhibition of elementary

[1] Department of Logic and Methodology of Science, Faculty of Arts, Comenius University, Bratislava, E-mail: sedlar@fphil.uniba.sk

[2] Department of Applied Informatics, Faculty of Mathematics, Physics and Informatics, Comenius University, E-mail: sefranek@ii.fmph.uniba.sk

cognitive capabilities. According to Kováč [15], biological evolution is a progressing process of knowledge acquisition.

The analysis of behavior of apes, dogs and other animals with an observable level of cognitive abilities leads to conclusions that the animals are able to reason and that they have knowledge about the external world.

Living agents (e.g., dogs, apes, and sometimes also people) observe results of their own actions or of actions of other agents. They distinguish success or failure of actions and learn on the basis of such observations etc.

Everyday behavior of living agents forces some kinds of reasoning and of knowledge acquisition. External conditions and criteria are crucial for the successful achieving of goals, for confirmation or supporting of the acquired knowledge. Correctness and usefulness of reasoning is tested with respect to external conditions.

The theoretical stance emphasizing the role of external conditions, when truth of a piece of knowledge and correctness of an act of reasoning is considered, shall be called *externalism* in this paper. Of course, knowledge and reasoning are supported by some mental and neural processes. The point of view, which abstracts from the role of these processes may be called *epistemological*. The basic ideas of our understanding of externalism and the epistemological point of view are discussed below.

### 2.1 Cognition: Belief, Confirmation and Falsification

The behavior of agents in some new, not completely known conditions and tasks is usually a process of trial and error. Agents observe responses of the external environment to actions, learn from the results of this process and fix the corresponding knowledge or beliefs (we do not distinguish here between knowledge and belief, even if it is possible and sometimes also necessary).

An epistemological translation of the paragraph above is that agents confirm or falsify their beliefs, while the confirmation or falsification takes the external environment into account.

### 2.2 Cognition and Reasoning

Let us discuss the role of reasoning within the tasks of confirmation and falsification. There are very simple forms of confirmation and falsification, sometimes based on the elementary level of reflexes. We are interested here only in the role of reasoning in those tasks. Importantly, confirmation and falsification may be represented as processes of formulating arguments and counterarguments. Obviously, reliable criteria enabling to decide if an argument is defeated by another argument are needed. Only external, intersubjective criteria are relevant as a tool of evaluation of defeats from the epistemological point of view.

To sum up: truth is a crucial attribute of contents of cognition (of beliefs); confirmation and falsification are used to evaluate truth of beliefs; reasoning, which respects some reliable and intersubjective criteria, is a tool of confirmation and/or falsification.

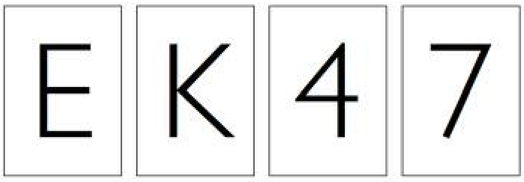
## 3. Reasoning: A Psychological Point of View

A natural outcome of our externalist and epistemological standpoint is the view that formal logic is rather important with respect to the understanding of human cognition. However, this stance has been challenged.

This section discusses two well-known psychological experiments, considered to be of great relevance to questions concerning the psychology of human reasoning. Both may be used to argue that formal logic does not account for the ways humans actually reason.

### 3.1 The Selection Task

The selection task, also known as Wason's task or Wason's selection task (see Wason [22],[23]), is constructed as follows. Subjects are shown four cards with numbers on one side and letters on the other. For example:




Subjects are then confronted with the following rule:

"If there is a vowel on one side, then there is an even number on the other side" (1)

Their task is to identify the cards which it is necessary to turn if one has to confirm or falsify the rule. In other words, they have to point to the cards (an only those cards) which have to be turned in order to settle the question if the rule applies to the displayed cards or not.

Surprisingly, the majority of subjects select E and 4 (usually around 45%) or E alone (35%). Only 5% of the subjects select what seems to be the correct answer, viz. E and 7 (data source: Stenning and van Lambalgen [21, p. 46]).[3]

It is easy to see the results as implying that actual human reasoning proceeds quite differently than by the rules of logic. More on this in section 3.3..

### 3.2 The Suppression Task

The suppression task (Byrne [8]) shows that additional premises may change subjects' inferences. For example, the inference

If she has an essay to write, she will study late in the library

[3] One could reason as follows. In order to confirm the rule, it is necessary to rule out the possibility that it does not apply. The rule does not apply if there is a card with a vowel on one side and an odd number on the other. Therefore, it is necessary to check if there are cards of this sort. However, only E and 7 can be the possible falsifiers, therefore it is necessary to turn these.

She has an essay to write

Therefore: She will study late in the library

is made by 90% of the subjects: they use modus ponens correctly when $A$ and $A \to B$ are the only premises. (Data source: [21, p. 181]) Now an additional premise is added, e.g.:

If she has an essay to write then she will study late in the library
*If the library stays open then she will study late in the library*
She has an essay to write

This changes the situation dramatically. Only 60% of the subjects make the modus ponens inference and conclude "She will study late in the library". This means that many subjects do not make the valid inference when an additional premise is present: many subjects do not use modus ponens when the premises are $A \to B$, $C \to B$ and $A$). The experimental data has been used as a basis for denying the relevance of logic for human reasoning (See Oaksford and Chater [20] for example, where a probabilistic approach is advocated).

### 3.3 Comments

The experiments discussed in the previous section seem to suggest that even the simplest inference rules of propositional logic are not followed by a significant portion of subjects involved in reasoning tasks. Does this mean that logic is not relevant with respect to the understanding of actual human reasoning? (Or at least not as relevant as it was assumed to be?)

This subsection provides a preliminary answer. We put forward several straightforward remarks concerning the design decisions and the interpretations of results of both experiments.

First, the selection task is not a reasoning task, but a combinatorial task. We should explain the difference. Of course, some nontrivial reasoning is required for writing an essay or designing a hat or selecting an optimal option from a set of options. But logic is not and cannot be interested in a detailed description of such tasks (and many similar or substantially different tasks). Reasoning relevant from the logical point of view should be carefully described. Logic aims at *characterizing entailment*: if a set of premises is given, what is a correct conclusion?[4] A derivation of conclusions from premises is understood usually as a reasoning task.

Now, back to the selection task – it is a combinatorial task, but with insufficient, incomplete, unclear input. It is implicitly assumed that experimental subjects should know that the words "if .... then ...." are interpreted as material implication. Of course, this is an unjustified assumption.

As regards the suppression task, capability to apply modus ponens is tested. The essentially lower ratio of applications of modus ponens in the second case is interpreted as evidence that people do not use modus ponens automatically, in each situation, but

[4]We will characterize some other domains of logic in Section 5.

human reasoning is dependent on the content and the context. This is true, but it is not an argument relevant with respect to logic and to applications of logic in human reasoning. Each logician or mathematician uses modus ponens only when true premises are given. In the second case of the suppression task there was a symptom that the premises may be not true. Consequently, the behavior of experimental subjects not applying modus ponens is quite rational (and, it could be said that the experimental subjects, who applied modus ponens blindly, did not reason carefully).

Note that the behavior of some experimental subjects was non-monotonic: they applied modus ponens to a subset of premises, but not to its superset. There are non-monotonic logical systems taking into account symptoms that some premises could be questioned and, therefore, some previous consequences cannot be derived anymore.

Let us return back to psychological experiments. Consider the following "experiment". Suppose that some experimental subjects should solve the following task. There are seven sheep on the meadow under the forest in the morning. Two sheep walked off the meadow later. How many sheep remained on the meadow?

97% of experimental subjects responded correctly. In the second round of the experiment, additional information is presented: Visibility is rather poor under the forest in the morning.

After receiving the additional information 34% of experimental subjects said that 5 sheep remained on the meadow, 12 % that 4 or 6 remained and the rest that the task does not have a solution.

The conclusion of the experimenter was as follows: Counting is viewed by arithmetic as a content-independent procedure applied impartially and uniformly to every problem regardless of the content involved. Hence, one could reason, it is not an appropriate tool for humans in real conditions.

Of course, nobody designed such an experiment. Everybody knows that truths of arithmetic do not depend on abilities of people to count or on conditions, where arithmetic could or could not be applied.

The same holds for logic. Validity of logic does not depend on abilities of people or on conditions of applicability of logic to reasoning tasks. Abilities of people, success or failure of human behavior are interesting from the psychological point of view. However, such aspects are not interesting from the logical point of view.

### 3.4. Logic and Actual Reasoning

The mainstream notion of logic sees its subject-matter as rather distinct from "actual human reasoning". It is not the task of logicians to study mental processes inside humans when they reason. They even do not have to rely on popular beliefs about the correctness of specific inferences.

According to the mainstream view (which has its roots in Frege's anti-psychologistic attitude towards logic and the foundations of mathematics), logic is seen as dealing with the *criteria of correctness* of inferences. Correctness is often specified as *truth-preservation*. Given a set of premises, which propositions cannot fail to be true in case all the premises are true? In other words, logic deals with *consequence*. Its task is to come up with an appropriate *definition* of consequence and with its matching formal *models*. These usually come in form of a formal language with appropriate semantics, together with a consequence relation. This is a relation between sets of sentences of the formal language and sentences, defined either syntactically (in terms of inference rules) or semantically (in terms of the semantic structures).

Philosophically speaking, this picture is sometimes summed up by saying that logic is *normative*. The task of logicians is to come up with formal models of consequence which in turn prescribe what is to count as correct inference. Consequently, any actual inference deviating somehow from the pattern prescribed by the formal model is deemed incorrect.

We think that the mainstream view is correct in its emphasis of formal models of inference. However, the idea that "Logic" is somehow superior to "actual reasoning" is too simplistic.

First, there is no such thing as "Logic". Some inferences that are correct from the viewpoint of classical logic are not correct from the viewpoint of intuitionistic logic, for example. (The view sketched here is sometimes called 'logical pluralism', see Beall and Restall [4])

Second, the factual evidence that comes from the history of modern logic (in the $20^{th}$ century) is overwhelming. Simply said, most of the "non-classical" logics that emerged during the previous century (and that keep on emerging until this day), have their *raison d'être* deeply rooted in a felt discrepancy between the "predictions" of a formal model of inference (usually classical logic) and (intuitions about) actual inferences.

Modal logics are a good example. Almost every beginning logic student finds the properties of material implication somehow awkward. The inference from $p$ to $q \to p$, deemed correct by classical propositional logic, is seen as suspicious. To be more specific, the "if …, then …" (or "… implies…") of natural language behaves differently than the "$\to$" of classical propositional logic. Modern modal logic emerged from the need to provide a more appropriate formal model of "if…, then…". C. I. Lewis (see Lewis [16], Lewis and Langford [17]), the key figure of its early modern history, thought that adding necessity is sufficient: 'Necessarily ($p \to q$)' was seen by him as the correct formal rendering of "$p$ implies $q$". However, this formal model has its own discrepancies and soon more refined models were suggested, viz. the various relevance logics (Anderson and Belnap [2],[3] and Mares [19]).

This ambition of modern logicians to "keep up" with the intuitions about actual reasoning and usage of the "logical words" has many examples, viz. conditional logics, non-monotonic logics, etc.

To sum up, the relation of logic to actual reasoning is not as simple as it may seem. First, logic is not to be thought of as a single set of 'correct' rules of inference. A more appropriate view of logic is to see it as a discipline aiming at providing formal models of inference and inference-related concepts. Second, these models are strikingly diverse and most of them were born of the need to model actual reasoning more flexibly and appropriately.

An important consequence of this viewpoint is that the relevance of logic to actual reasoning cannot be conclusively refuted by pointing out that a particular logical system does not fit in with intuitions or experimental data. There is always the possibility of providing a more appropriate system.

## 4. Logic Strikes Back

An important recent defense of the relevance of logic for cognitive science is Stenning and van Lambalgen [21]. Their strategy is to assess the importance of subjects' interpretations of the reasoning tasks, such as the Selection task or the Suppression task („reasoning *to* an interpretation").

Their claim is that after the interpretation has been settled, one may proceed to a formal model of the subjects' responses ("reasoning *from* an interpretation"). Stenning and van Lambalgen argue for the prominence of non-monotonic logics as a model of reasoning. Special attention is devoted to an interpretation and formalization of conditional sentences (rules) with exceptions.

## 5. Logic and Human Reasoning

This section outlines the reason why the logical point of view and methods of logic are indispensable for the understanding of reasoning, and hence for the understanding of human cognition. First, a possible way how logic as a scientific field evolved from sophisticated human reasoning is sketched. After that some logical systems are discussed from the viewpoint of relevance for understanding of reasoning and cognition. The systems are presented in a somewhat sketchy manner, as a thorough exposition is not the aim of this chapter.

### 5.1. Human reasoning and logic

We shall argue that a logical representation of reasoning is a natural result of the cultivation of human cognitive capabilities and of the attempts to understand and describe our reasoning.

Our starting point is counter-argumentation via the search for counterexamples. If somebody wants to show that the arguments of his opponent are wrong, she may try to construct a similar flow of claims (sentences) which leads from true premises to a false conclusion. There is an analogy to attempts to falsify general statements. An obvious procedure is to find a special case for which the general statement is not true.

There is a nice example in the history of human thought. Socrates mastered the art of counter-argumentation and constructing of counterexamples as a tool of rational dialogue, most importantly as a tool of uncovering the falsity of someone's beliefs, as a tool of supporting our knowledge via arguing against unsupported claims. His influence leaded through Plato to an invention of a logical system by Aristotle.

The step from counter-argumentation to logic is simple. First, we emphasize that the construction of counterexamples entails a shift from the content of sentences to their form – counterexamples are of the same form as the attacked sentences. Second, the focus is shifted from the construction of counterarguments to finding ways of reasoning which are immune from counterarguments. In other words, the attention of (not only Aristotelian) logic was and is focused on *truth-preserving* schemes of reasoning. Remember the well known syllogism: if each $A$ is $B$ and each $B$ is $C$, then each $A$ is $C$. It is impossible to find a counterexample (a substitution of some notions, names of classes ) such that each $A$ is $B$, each $B$ is $C$, but there are some $A$, which are not $C$. On the other hand, you can find a counterexample to the following form of reasoning: if some $A$ are

*B*, some *B* are *C*, then some *A* are *C*. The first scheme of reasoning preserves truth (it leads necessarily from true premises to a true conclusion). The second scheme is obviously not truth-preserving.

An important feature of correct human reasoning is an ability to preserve truth of basic postulates, facts and starting points in the flow of reasoning to the truth of consequences. A fundamental relevance of logic for cognitive science is based on that observation.

We have to reflect the development of logic from Aristotelian times to the state, where a rich variety of logical systems and ways how to do logic (Makinson [18]) is available. Schemes of reasoning uncovered by a logical system preserve truth, if logical constants (each, some, if – then, possibly, etc.) are understood in the way specified by the logical system. Reasoning to an interpretation, as understood by (Stenning and van Lambalgen [21]) is a procedure leading to a selection of an appropriate understanding of logical constants for a given reasoning task. Moreover, a need to specify and to model in an abstract way new logical constants or some new meaning of a logical constant leads often to a new logical system and to a new option for a reasoning to an interpretation.

However, the characterization of the variety of reasoning procedures and styles is not exhausted by simple truth preservation. Different types of schemes of truth-preserving reasoning provide a characterization of different forms of *deduction*. We have to mention also *hypothetical* reasoning also.

Fortunately, (at least some of) people reason even if they do not have only true premises at their disposal. This kind (more precisely, a class of kinds) of reasoning is studied intensively in artificial intelligence. Non-monotonic reasoning and defeasible reasoning are the terms used in artificial intelligence. We will use the term 'hypothetical reasoning' and note that it can be (and is) described in many different ways. We sketch only a simple characterization here.

When people reason hypothetically, they consider a set of defeasible assumptions. In general there are some incompatible assumptions in the set. Some assumptions attack other assumptions (via their consequences). Usually, it is not possible to speak about the correctness of an isolated assumption. It is more productive to consider sets of assumptions and to check whether they are defended against the attacks of some (counter)assumptions. A conflict-free set of assumptions *S* is *admissible* if it counterattacks each attack against each member of *S*. This is the basic idea of Dung [10], where this notion was introduced precisely. In Bondarenko et al. [7] it was adapted for assumption-based frameworks and applied to a characterization of default reasoning in the frame of various non-monotonic formalisms.

Different kinds of non-monotonic logic, defeasible logic, argumentation frameworks, logic programming etc. study hypothetical reasoning and various types of sets of admissible assumptions. Some argumentation semantics, based on notions of conflict-freeness and admissibility are discussed later in this section.

We conclude this subsection as follows. There are two important features of human reasoning – preservation of truth and accepting of admissible sets of assumptions. The *logical point of view* can be characterized (at least for the aims of this chapter) as focused on truth-preservation or on admissibility of assumptions (arguments ). *Methods* of logic consist in an abstraction from the content of pieces of knowledge or sets of sentences, in the construction of some symbolic, formal languages[5], which enable an abstract and general treatment of a kind of reasoning.

It is important to note that the method enables a highly detailed description of reasoning and that thanks to this level of details it is possible to construct computational models of reasoners and to implement them in real applications.

Epistemological point of view was characterized by an abstraction from mental processes and by an emphasis on reliable, intersubjective criteria. Logical point of view and methods of logic contribute to understanding and modeling correct reasoning in accordance with the principles mentioned above.

## 5.2 Basic logical systems

We are aiming to show how a cognitive stance may influence a construction of a logical system.

The classical two-valued logic is based on a platonic view of the world: individuals have or have not some properties, a situation or an event occurs or does not occur, a sentence is true or false. Tertium non datur (the law of excluded middle) is a logical expression of this basic attitude. Similarly, contradictions are not allowed. Thus, a proposition that there is an object which some property may be proved if it is demonstrated that an assumption about the non-exisetnce of such an object leads to a contradiction. This stance is sometimes characterized by the slogan that logic is a set of features of the world and the (!) correct reasoning consist in discovering those features.

Constructivist logic evolved as an opposition to the kind of logic characterized in the previous paragraph. Existence of an object satisfying a property can be proved only if the object with that property is constructed. According to a branch of constructivism called intuitionism, methods of construction are based on natural human intuition. Thus, correct reasoning consist in following given capabilities of our mind. Of course, constructivist logical systems are sharply separable from the not too clear philosophical motivations and the difference with respect to classical logic may be characterized precisely.

Another stream of logical systems deviating from the classical logic enlarges the set of truth values. Truth and falsity are not the exclusive values, the third value was introduced first. Systems with infinite sets of truth values were constructed. Fuzzy logics, which emphasize that the borders between classes, properties etc. may not be sharp, are in fact multi-valued logics.

Finally, we mention the Kripkean semantics. This style of semantics provides a characterization of such propositions, where a direct assignment of a truth value is not appropriate. Epistemic logic, presented below, is one example.

## 5.3 Epistemic logic

[5] An objection against logic is that it constructs some strange artificial languages. We hope that it is only a marginal stance. Nobody criticizes physics or engineering because of their use of an artificial language. Similarly, logic discovers fundamental knowledge thanks to an art of abstraction and focusing on principles.

One of the most prominent current approaches to modeling information and cognition is epistemic logic. Epistemic logic dates back to the seminal works of von Wright [24] and Hintikka [14]. This subsection offers a sketch of its basics.

The language of epistemic logic extends the language of classical propositional logic by a family of knowledge operators $K_i$, where $i$ ranges over some set of agents $G$: $K_i\, p$ is read 'agent $i$ knows that $p$'.

Epistemic models for a set of agents $G$ are structures $M = (W, \{R_i\}_{i \in G}, V)$, where $W$ is a non-empty set, every $R_i$ is a binary relation on $W$ and $V$ is a valuation, i.e. a function from the set of propositional atoms to subsets of $W$. Informally, $W$ is thought of as the set of epistemic alternatives or possible worlds. However, it is usual to refer to them in a more neutral manner as 'points'.

Next, $R_i$ is an epistemic indistinguishability relation for the agent $i$: $R_ixy$ iff $i$ cannot distinguish between points $x$, $y$. To be more specific, $i$ cannot distinguish between $x$ and $y$ if she does not have access to information that would render one of the points as obviously incorrect. For example, if I do not know whether it is sunny in London, then I cannot distinguish between any $x$ containing the fact that it is sunny in London and any $y$ containing the fact that it is not sunny there. In most applications, the indistinguishability relations are assumed to be equivalence relations, i.e. reflexive, symmetric and transitive.

Truth of formulas is relative to points: most importantly, $K_iA$ is true at a point $x$ iff $A$ is true at every $y$ such that $R_ixy$. Hence, $i$ knows that $A$ iff $A$ holds at every epistemic alternative. This is in line with our intuitions – if I know that $A$, then points with non-$A$ are obviously not sound epistemic alternatives.

Epistemic logic clarifies several somewhat involved scenarios such as the muddy children puzzle and is often used in computer science (see Fagin et al. [11]). One of its positive features is the ability to represent knowledge of agents about the knowledge of other agents in a clear way.

However, epistemic logic also has a number of counterintuitive features. For example, if a formula $A$ holds at every point in some model $M$ then $K_iA$ also holds in every point in $M$. (The reason is simple: the set of '$i$-reachable' points from any given $x$ is obviously a subset of $W$). Read informally, this means that every agent $i$ knows every valid formula! Moreover, the formula $K_i(A \rightarrow B) \rightarrow (K_iA \rightarrow K_iB)$ is valid in every $M$. In other words, knowledge is closed under modus ponens. Together with the knowledge of valid formulas, this entails that knowledge is closed under valid implications. As a special case, knowledge is closed under classical consequence.

However, it is obvious that this is an idealized situation. In many situations agents do not know every consequence of their knowledge. For example, they might didn't perform the needed inference steps or they lack the computational resources to do so. This obvious discrepancy between our intuitions about the knowledge of real agents and the 'predictions' of epistemic logic is known as *the logical omniscience problem*.

The standard answer is to distinguish between *implicit* and *explicit* knowledge. Explicit knowledge is seen as a body of consciously accepted and confirmed information. On the other hand, implicit knowledge is the body of logical consequences of explicit knowledge. It is acknowledged that the $K_i$ operators represent implicit knowledge.

However, the issue of modeling explicit knowledge remains interesting. The literature offers several approaches (see Fagin et al. [11, ch. 9]).

## 5.4 Dynamic Epistemic Logic

The kind of epistemic logic described in the previous section is often described as being static. It models information states of agents by the indistinguishability relation, but these are information states at a given time.

However, an important feature of knowledge and cognition is its *dynamics*. We often revise our beliefs in the face of new evidence, or supplement our information as a result of observation and communication.

Modern logic offers several formal models of information dynamics. An important contribution is the *belief revision theory*, see Alchourrón, Gärdenfors and Makinson [1]. It is a refined model of belief change with several distinguished modes of change. The first one is simple *expansion*: sometimes we add to our beliefs new information that does not conflict with our previous beliefs. The second one is *contraction*: sometimes we abandon our beliefs for various reasons. The third, and perhaps the most important one, is *revision*: sometimes new beliefs have to be added which contradict some of the previous beliefs. Now the problem is to restore consistency. Which beliefs is it best to abandon? The belief revision theory offers an interesting answer, which is, however, beside the scope of this chapter.

A different model of information dynamics is the *public announcement logic* (see van Ditmarsch, van der Hoek and Kooi [9, ch. 4]. This is an extension of the basic epistemic logic of section 5.3. Importantly, the basic epistemic language is extended by a modality $[A]$ for any formula $A$. Now the formula $[A]B$ is read 'After a truthful public announcement of $A$, $B$ is the case.' Semantically, $[A]B$ is true at a point $x$ iff $A$ is true at $x$ and $B$ is true at $x$ with respect to a model where every point that does not make $A$ true has been deleted.

Public announcement logic is a simple formal model of communication and public observation with many interesting applications. For more information on the formal models of epistemic dynamics, see van Benthem [5].

## 5.5 Admissibility semantics

This subsection provides a cursory view on a formalization of defeasible reasoning. The formalization is not our primary goal. More interesting is to show that a formal, logic-based approach to a description of hypothetical reasoning is possible and fruitful. We describe a very simple, but elegant abstract argumentation framework by Dung [10].

An abstract argumentation framework $AF$ is a pair $(A, R)$, where $A$ is a set (of arguments) and $R$ is a binary relation on $A$. If a pair of arguments $(a, b)$ is in $R$, it is said that $a$ attacks $b$. Notice that nothing is supposed about the structure of arguments. Similarly, no details about the attack relation are given.

Dung accepted an essential decision – a status of an argument may be specified reasonably only with respect to a set of arguments. Hence, an argument $a$ is *acceptable* with respect to a set of arguments $S$, if and only if for each argument $b$ attacking $a$ there is an argument $c$ in $S$ attacking $b$. A set of arguments $S$ is *conflict-free* iff there is no pair $a$, $b$ in $S$ such that $a$ attacks $b$. We already know that a conflict-free set of arguments $S$ is *admissible* iff each attack of an argument $b$ against an argument $a$ in $S$ is counterattacked by $S$, i.e., there is an argument $c$ in $S$ such that $c$ attacks $b$.

Some argumentation semantics (mappings from AF to sets of sets of arguments) are presented below. A conflict-free set of arguments *S* is a *preferred extension* of *AF* iff it is a maximal (with respect to the subset relation) admissible set of arguments from *AF*. *S* is a *stable extension* of *AF* iff each argument not in *S* is attacked by an argument in *S*. *F*, a *characteristic function* of *AF*, assigns to a set of arguments *S* the set of all arguments acceptable with respect to *S*. The (only) *grounded extension* of *AF* is the least (with respect to the subset relation) set of arguments *G* such that $F(G) = G$. It is said that *G* is the least fixed point of *F*. The existence of the least fixed point of *F* follows from the monotony of *F*.

Other semantics of abstract argumentation frameworks were proposed since Dung's seminal paper and logical research in argumentation proceeded also from abstract frameworks to structured frameworks, where the structure of arguments is interesting and also other logical aspects of argumentation are considered.

It is important to notice that logical research of argumentation abstracts from such problems or claims as: persuade your partner in a dialogue that your opinion is his opinion etc. Attention of logics is focused only on arguments and their relations.

Other, yet more important remark is the following. Semantics of other non-monotonic formalisms can be expressed in terms of argumentation semantics. Let us consider logic programs under the stable models semantics (Gelfond and Lifschitz [12] ). Nowadays, the term ‘answer set semantics’ is used more often and answer set programming (ASP) became a leading paradigm in implementation and theoretical research of knowledge representation and reasoning in artificial intelligence. It was shown already in [10] that logic programs may be represented as argumentation frameworks and some semantics of logic programs correspond to argumentation semantics. Most importantly, from the viewpoint of ASP, stable models correspond to stable extensions. An early extensive work studying relations of non-monotonic formalisms and argumentation semantics is Bondarenko et al. [7].

In order to close this subsection: it was shown that hypothetical (defeasible, non-monotonic) reasoning may be viewed as an argumentation framework, where some assumptions play a role of arguments, conflicts between sentences are considered as attacks between arguments and solutions of conflicts are specified by some semantics. However, we did not choose the argumentation frameworks with an intention to present it as a leading paradigm in the research of hypothetical reasoning. A final remark - hypothetical reasoning is closely connected to a representation of dynamic aspects of knowledge and, consequently, to belief change (updates and revisions).

## 6. Conclusions

An activity highly relevant for cognitive science is, e.g., to present a field of knowledge, which is assessed as unquestionable and, moreover, to develop a language and a method supporting that goal. An unprecedented intellectual boom connected to the mathematical logic research in the beginning of $20^{th}$ century was oriented towards that kind of goals – to provide firm and unquestionable foundations of mathematical knowledge.

It is well known that the deep results by Goedel, Turing, Tarski and others rendered this ambition hopeless. However, a more modest characterization given by Barwise and Etchemendy [6], who claim that mathematical logic is an idealized presentation and communication of mathematical results seems to be interesting from the cognitive science point of view, too.

It was emphasized repeatedly in this chapter that real-world stimuli contributed to the development of new systems and kinds of logic. A role of artificial intelligence was noticed in this context, too. We shall conclude by an interpretation of the role of logic in computer science (Halpern et al. [13]) from the viewpoint of cognitive science. Logic is proven to be an appropriate and effective tool for development of theories and constructions in computer science (program specification and verification, programming languages research, databases research, complexity theory, multi-agent systems, automated design verification, knowledge representation etc.). This fact is contrasted with respect to the essentially less important role of mathematical logic in contemporary mathematics. An application of languages and systems intended for idealized modeling of some aspects of the world and some kinds of entailment relation to a successful description of domains interesting for computer science and for reasoning about those domains could be interpreted as a success story from the cognitive science point of view; certainly, cognitive capabilities are needed for such applications.

**Acknowledgments:** Work of the first author was supported by the grant 'Semantic models, their explanatory powers and applications' (VEGA 1/0046/11). The work of the second author was supported by the grant VEGA 1/1333/12.

## References

[1] Alchourrón, C.E., Gärdenfors, P. and Makinson, D.: On the Logic of Theory Change. *Journal of Symbolic Logic* **50** (1985), pp. 510 – 530.

[2] Anderson, A. R., Belnap, N. D.: *Entailment*, volume 1. Princeton University Press, Princeton, 1975.

[3] Anderson, A. R., Belnap, N. D.: *Entailment*, volume 2. Princeton University Press, Princeton, 1992.

[4] Beall, J. C., Restall, G.: *Logical Pluralism*. Clarendon Press, Oxford, 2006.

[5] van Benthem, J.: *Logical Dynamics of Information and Interaction.* Cambridge University Press, Cambridge, 2011.

[6] Barwise, J., Etchemendy, J.: Computers, visualization, and the nature of reasoning. In T.W. Bynum and J. H. Moor (Eds.) *The Digital Phoenix: How Computers are Changing Philosophy*. Blackwell, Oxford, 1998, pp. 93 – 116.

[7] Bondarenko, A., Dung, P. M., Kowalski, R. A. Toni, F.: An abstract, argumentation-theoretic approach to default reasoning. *Artificial Intelligence* **93** (1997), pp. 63 – 101.

[8] Byrne, R. M. J., Suppressing valid inferences with conditionals. *Cognition,* **31** (1989), pp. 61 – 83.

[9] van Ditmarsch, H., van der Hoek, W., and Kooi, P.: *Dynamic Epistemic Logic*. Springer, Dordrecht, 2008.

[10] Dung, P. M.: On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and *n*-person games. *Artificial Intelligence* **77** (1995), pp. 321-357.

[11] Fagin, R., Halpern, J. Y., Moses Y. and Vardi, M. Y.: *Reasoning about Knowledge*. MIT Press, Cambridge (MA), 1995.

[12] Gelfond, M and Lifschitz, V.: The Stable Model Semantics for Logic Programming. *ICLP/SLP* (1988), pp. 1070 – 1080.

[13] Halpern, J.Y., Harper, R., Immerman, N., Kolaitis, P.G., Vardi, M.Y., Vianu, V.:On the unusual effectiveness of logic in computer science. *Bulletin of Symbolic Logic* **7** (2001) pp. 213 – 236.

[14] Hintikka, J.: *Knowledge and Belief: An Introduction to the Logic of the Two Notions*. Cornell University Press, Ithaca, 1962.

[15] Kováč, L.: Information and Knowledge in Biology: Time for Reappraisal. *Plant Signaling and Behavior*, **2** (2007), pp. 65 – 73.

[16] Lewis, C. I.: *Survey of Symbolic Logic.* University of California Press, Berkeley, 1918.

[17] Lewis, C. I., Langford, C. H.: *Symbolic Logic*. Century, London, 1932.

[18] Makinson, D.: Ways of doing logic: what was different about AGM 1985? *Journal of Logic and Computation* **13** (2003), pp. 3 – 13.

[19] Mares, E.: *Relevant Logic. A Philosophical Interpretation*. Cambridge University Press, Cambridge, 2004.

[20] Oaksford, M. and Chater, N.: The probabilistic approach to human reasoning. *Trends in Cognitive Sciences* **5** (2001), pp. 349 – 357.

[21] Stenning, K., van Lambalgen, M.: *Human Reasoning and Cognitive Science*, MIT Press, Cambridge (MA), 2008.

[22] Wason, P. C. Reasoning about a rule. *Quarterly Journal of Experimental Psychology,* **20** (1968), pp. 273 – 281.

[23] Wason, P. C.: Reasoning. In B.M. Foss (Ed.): *New Horizons in Psychology I.* Penguin, Harmondsworth, 1966.

[24] Von Wright, G. H.: *An Essay on Modal Logic*. North-Holland Publishing Company, Amsterdam, 1951.